MECHANICAL WORK, ENERGY, AND POWER

Dr. Kurtuluş Erinc Akdoğan

kurtuluserinc@cankaya.edu.tr
INTRODUCTION

- Variables related to **energetics** variable contains the most information
- Diagnostically, we have found joint mechanical **Powers** to be the most discriminating in all our assessment of pathological gait.
- Without them, we could have made erroneous or incomplete assessments that would not have been detected by EMG or moment-of-force analyses alone.
- Valid **mechanical work** calculations are essential to any **efficiency** assessments that are made in sports and work-related tasks.
Mechanical Energy and Work

- Mechanical energy and work have the same units (joules) but have different meanings.
- Mechanical **energy** is a measure of the state of a body at an instant in time as to its **ability to do work**.
- For example, a body which has 200 J of kinetic energy and 150 J of potential energy is capable of doing 350 J of work (on another body).
- Work is the measure of energy flow from one body to another, and time must elapse for that work to be done.
- If energy flows from body A to body B, we say that body A does work on body B; or
- muscle A can do **work** on segment B if **energy flows** from the muscle to the segment.
Internal versus External Work

- In a lifting task, the work rate on the external load might be 200 W, but the work rate to increase the energy of the total body by the source muscles of the lower limb might be 400 W. Thus, the sum of the internal and external work rates would be 600 W.

- A distinction is made between the work done on the body segments (called internal work) and the work done on the load (called external work).
Bicycle ergometry situation in which one subject (left) cycles in a forward direction and does work on a second subject who cycles in the reverse direction.

Both cyclists are not performing equal magnitudes of mechanical work.

The positive-work cyclist not only does external work on the negative-work cyclist but also does the internal work to move the limbs of both cyclists.
Resisting the reverse moving pedals with the leg muscles inducing lengthening, "negative work"

Pushing the pedals forward with the leg muscles inducing shortening, "positive work"
Positive work is work done during a concentric contraction, when the muscle moment acts in the same direction as the angular velocity of the joint.

If an extensor muscle moment is negative and an extensor angular velocity is negative, the product is still positive.
Negative Work of Muscles

- Negative work is work done during an eccentric contraction when the muscle moment acts in the opposite direction to the movement of the joint.

- When there is an extensor moment (negative) during a flexor angular change (positive), the product is negative.
Positive Work and Negative Work of Muscles

**Concentric Action**
- **Movement**
- **Positive Work**
  - Shortening against gravity

**Eccentric Action**
- **Movement**
- **Negative Work**
  - Lengthening against gravity
Muscle Mechanical Power

- During the initial extension, there is an extensor moment and an extensor angular velocity as the triceps do positive work on the forearm.

- During the latter extension phase, the forearm is decelerated by the biceps (flexor moment). Here the biceps are doing negative work (absorbing mechanical energy).

- Once the forearm is stopped, it starts accelerating in a flexor direction still under the moment created by the biceps, which are now doing positive work.

- Finally, at the end of the movement, the triceps decelerate the forearm as the extensor muscles lengthen; here, $P_m$ is negative.

\[
P_m = M_j \omega_j \quad \text{W}
\]

- $P_m$ = muscle power, watts
- $M_j$ = net muscle moment, N $\cdot$ m
- $\omega_j$ = joint angular velocity, rad/s
Mechanical Work of Muscles

- The product of power and time is work.
- If a muscle generates 100W for 0.1 s, the mechanical work done is 10 J.
- To calculate work done, power must be integrated over a period of time.

\[ W_m = \int_{t_1}^{t_2} P_m \, dt \quad \text{J} \]
Mechanical Work Done on an External Load

- Work is defined as the product of the force acting on a body and the displacement of the body in the direction of the applied force.

\[ dW = F \, ds \]
\[ W = \int_{0}^{S_1} F \, ds = FS_1 \]

- Power is the rate of doing work

\[ P = \frac{dW}{dt} = F \frac{ds}{dt} = F \cdot \overline{V} \]

\[ P = \text{instantaneous power, watts} \]
\[ \overline{F} = \text{force, newtons} \]
\[ \overline{V} = \text{velocity, m/s} \]

\[ P = FV \cos \theta = F_x V_x + F_y V_y \]

\[ \theta \] = angle between force and velocity vectors in the plane defined by those vectors

\[ F_x \] and \[ F_y \] = forces in \( x \) and \( y \) directions

\[ V_x \] and \[ V_y \] = velocities in \( x \) and \( y \) directions
Example

A baseball is thrown with a constant accelerating force of 100N for a period of 180 ms. The mass of the baseball is 1.0 kg, and it starts from rest. Calculate the work done on the baseball during the time of force application.

\[
S_1 = ut + \frac{1}{2}at^2
\]

\[
u = 0
\]

\[
a = \frac{F}{m} = \frac{100}{1.0} = 100 \text{ m/s}^2
\]

\[
S_1 = \frac{1}{2} \times 100(0.18)^2 = 1.62 \text{ m}
\]

\[
W = \int_0^{S_1} F \, ds = FS_1 = 100 \times 1.62 = 162 \text{ J}
\]
A baseball of mass 1 kg is thrown with a force that varies with time. The velocity of the baseball in the direction of the force is also plotted on the same time base and was calculated from the time integral of the acceleration curve. Instantaneous power to the baseball and the total work done on the baseball during the throwing period is shown.
Mechanical Energy Transfer between Segments

- Each body segment exerts forces on its neighboring segments, and if there is a translational movement of the joints, there is a mechanical energy transfer between segments.
- The product of $F_{j1}V_j \cos \theta_1$ is positive, indicating that energy is being transferred into segment 1.
- Conversely, $F_{j2}V_j \cos \theta_2$ is negative, denoting a rate of energy outflow from segment 2.
- Since $P_{j1} = -P_{j2}$, the outflow from segment 2 equals the inflow to segment 1.
- At the end of swing, for example, the swinging foot and leg lose much of their energy by transfer upward through the thigh to the trunk, where it is conserved and converted to kinetic energy to accelerate the upper body in the forward direction.
Potential Energy. Potential energy (PE) is the energy due to gravity and, therefore, increases with the height of the body above ground or above some other suitable reference.

\[ PE = mgh \quad J \]

- \( m \) = mass, kg
- \( g \) = gravitational acceleration, = 9.8 m/s\(^2\)
- \( h \) = height of center of mass, meters

Normally, ground reference is considered to be the lowest point that the body takes during the given movement.
Kinetic Energy

- There are two forms of kinetic energy (KE), that due to translational velocity and that due to rotational velocity.

\[
\text{translational KE} = \frac{1}{2}mv^2 \quad \text{J}
\]

\[
v = \text{velocity of center of mass, m/s}
\]

\[
\text{rotational KE} = \frac{1}{2}I\omega^2 \quad \text{J}
\]

\[
I = \text{rotational moment of inertia, kg \cdot m}^2
\]

\[
\omega = \text{rotational velocity of segment, rad/s}
\]
The energy of a body exists in three forms so that the total energy of a body is

\[ E_s = \text{PE} + \text{translational KE} + \text{rotational KE} \]

\[ = mgh + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \]
Calculate the energy of shank (between knee and ankle) and check whether it is equal to $E_{lg}$ given below.

$$E_{lg}(\text{frame 6}) = 20.5 \, \text{J}$$
Power Balance Within Segments

\[ \frac{dE_s}{dt} = P_{jp} + P_{mp} + P_{jd} + P_{md} \]
Example

- Carry out a power balance for the leg and thigh segments for frame 5, that is, deduce the dynamics of energy flow for each segment separately and determine the power dynamics of the knee muscles (generation, absorption, transfer):
hip velocities, \( V_{xh} = 1.36 \text{ m/s} \) \( V_{yh} = 0.27 \text{ m/s} \)
knee velocities, \( V_{xk} = 2.61 \text{ m/s} \) \( V_{yk} = 0.37 \text{ m/s} \)
ankle velocities, \( V_{xa} = 3.02 \text{ m/s} \) \( V_{ya} = 0.07 \text{ m/s} \)
leg angular velocity, \( \omega_{lg} = 1.24 \text{ rad/s} \)
thigh angular velocity, \( \omega_{th} = 3.98 \text{ rad/s} \)

leg segment reaction forces and moments,
\( F_{xk} = 15.1 \text{ N} \) \( F_{yk} = 14.6 \text{ N} \) \( F_{xa} = -12.3 \text{ N} \) \( F_{ya} = 5.5 \text{ N} \),
\( M_a = -1.1 \text{ N} \cdot \text{m} \) \( M_k = 5.8 \text{ N} \cdot \text{m} \)

thigh segment reaction forces and moments,
\( F_{xk} = -15.1 \text{ N} \) \( F_{yk} = -14.6 \text{ N} \) \( F_{xh} = -9.4 \text{ N} \) \( F_{yk} = 102.8 \text{ N} \),
\( M_k = -5.8 \text{ N} \cdot \text{m} \) \( M_h = 8.5 \text{ N} \cdot \text{m} \)

leg energy, \( E_{lg} \) (frame 6) = 20.5 J, \( E_{lg} \) (frame 4) = 20.0 J
thigh energy, \( E_{th} \) (frame 6) = 47.4 J, \( E_{th} \) (frame 4) = 47.9 J
1. Leg Power Balance

\[ \Sigma \text{powers} = F_{xk} V_{xk} + F_{yk} V_{yk} + M_k \omega_{lg} + F_{xa} V_{xa} + F_{ya} V_{ya} + M_a \omega_{lg} \]

\[ = 15.1 \times 2.61 + 14.6 \times 0.37 + 5.8 \times 1.24 - 12.3 \]

\[ \times 3.02 + 5.5 \times 0.7 - 1.1 \times 1.24 \]

\[ = 44.81 + 7.19 - 33.3 - 1.36 \]

\[ = 17.34 \text{ W} \]

\[ \frac{\Delta E_{lg}}{\Delta t} = \frac{20.5 - 20.0}{0.0286} = 17.5 \text{ W} \]

balance = 17.5 - 17.34 = 0.16 W
2. Thigh Power Balance

\[ \sum \text{powers} = F_{xh}V_{xh} + F_{yh}V_{yh} + M_h\omega_{th} + F_{xk}V_{xk} + F_{yk}V_{yk} + M_k\omega_{th} \]
\[ = -9.4 \times 1.36 + 102.8 \times 0.27 + 8.5 \times 3.98 \]
\[ - 15.1 \times 2.61 - 14.6 \times 0.37 - 5.8 \times 3.98 \]
\[ = 14.97 + 33.83 - 44.81 - 23.08 \]
\[ = -19.09 \text{ W} \]

\[ \frac{\Delta E_{th}}{\Delta t} = \frac{47.4 - 47.9}{0.0286} = -17.5 \text{ W} \]

balance = \(-17.5 - (-19.09)\) = 1.59 W
The power flows are summarized in Figure 6.20 as follows: 23.08W leave the thigh into the knee extensors, and 7.19W enter the leg from the same extensors. Thus, the knee extensors are actively transferring 7.19W from the thigh to the leg and are simultaneously absorbing 15.89 W.

There is power transfer from the thigh to the leg at a rate of 7.19W through the quadriceps muscles plus passive flow across the knee of 44.81 W.