KINETICS: FORCES AND MOMENTS OF FORCE

Dr. Kurtuluş Erinç Akdoğan

kurtuluserinc@cankaya.edu.tr
BIOMECHANICAL MODELS

- **Kinematics:** analysis of movement
- **Kinetics:** The study of forces causing the movement

Knowledge of forces is necessary for an understanding of the cause of any movement.

Forces can be measured **directly** by implating transducers into muscle. Forces can be calculated **indirectly** using readily available kinematic and anthropometric data.

The process by which the reaction forces and muscle moments are calculated is called **link-segment modeling**.

This prediction is called an **inverse solution** and is a very powerful tool in gaining insight into the net summation of all muscle activity at each joint.

Such information is very useful to the coach, surgeon, therapist, and kinesiologist in their diagnostic assessments.

The effect of training, therapy, or surgery is extremely evident at this level of assessment, although it is often obscured in the original kinematics.
Link-Segment Modeling

Kinematic data → Joint reaction forces

Kinetic data → Net muscle moments

Anthropometric data → Kinetic and potential energy

\[ M_{\text{muscle}} = M_{\text{quads}} - M_{\text{ham}} \]
Link-Segment Modeling

- Joints are replaced by hinge (pin) joints and segments are replaced by masses and moments of inertia located at each segment’s center of mass.
- The segment masses $m_1$, $m_2$, and $m_3$ are considered to be concentrated at points.
- The distance from the proximal joint to the mass centers is considered to be fixed, as are the length of the segments and each segment’s moment of inertia $I_1$, $I_2$, and $I_3$ about each COM.
Forces Acting on the Link-Segment Model

- **Gravitational Forces.** The forces of gravity act downward through the COMs of each segment and are equal to the magnitude of the mass times acceleration due to gravity (normally 9.8m/s²).

- **Ground Reaction or External Forces.** Any external forces must be measured by a force transducer. Such forces are distributed over an area of the body (such as the ground reaction forces under the area of the foot). In order to represent such forces as vectors, they must be considered to act at a point that is usually called the center of pressure (COP). A suitably constructed force plate, for example, yields signals from which the COP can be calculated.

- **Muscle and Ligament (bağ doku) Forces.** The net effect of muscle activity at a joint can be calculated in terms of net muscle moments. If a cocontraction is taking place at a given joint, the analysis yields only the net effect of both agonist and antagonistic muscles. At the extreme range of movement of any joint, passive structures such as ligaments come into play to contain the range.
Joint Reaction Forces and Bone-on-Bone Forces

- The three forces described in the preceding sections constitute all the forces acting on the total body system itself.

- However, our analysis examines the segments one at a time and, therefore, must calculate the reaction between segments. A free-body diagram of each segment is required.
Joint Reaction Forces and Bone-on-Bone Forces

- The original link-segment model is broken into its segmental parts.
- For convenience, we make the break at the joints and the forces that act across each joint must be shown on the resultant free-body diagram.
- This procedure now permits us to look at each segment and calculate all unknown joint reaction forces.
Joint Reaction Forces and Bone-on-Bone Forces

- In accordance with Newton’s third law, there is an equal and opposite force acting at each hinge joint in our model.

- For example, when a leg is held off the ground in a static condition, the foot is exerting a downward force on the tendons and ligaments crossing the ankle joint. This is seen as a downward force acting on the leg equal to the weight of the foot.

- Likewise, the leg is exerting an equal upward force on the foot through the same connective tissue.
Free body diagram

[ simplified drawing of a mechanical system, isolated from its surroundings, showing all forces vectors and torques ]

1. Particle (or point-mass) free body diagram – assumed that kinematics of object can be represented by its center of mass and its linear movement (e.g., parabolic path of the center of mass of a long jumper in flight) [Note that there is no angular motion associated with a point mass.]
Free body diagram (continued)

2. Segmental - can represent the human body mechanically as a linked system of rigid segments moving about an axes of rotation through joints

\[
\begin{align*}
mg &= \text{wt.} \\
\mathbf{f}_{xp} &\quad \mathbf{f}_{yp} \\
\mathbf{a}_x &\quad \mathbf{a}_y \\
\mathbf{a}_r &\quad \alpha \\
\mathbf{M}_d &\quad \mathbf{M}_p
\end{align*}
\]
Considerable confusion exists regarding the relationship between joint reaction forces and joint bone-on-bone forces. The latter forces are the actual forces seen across the articulating surfaces and include the effect of muscle activity and the action of ligaments. Actively contracting muscles pull the articulating surfaces together, creating compressive forces and, sometimes, shear forces. In the simplest situation, the bone-on-bone force equals the active compressive force due to muscle plus joint reaction forces. Case 1 has the lower segment with a weight of 100N hanging passively from the muscles originating in the upper segment. The two muscles are not contracting but are, assisted by ligamentous tissue, pulling upward with an equal and opposite 100 N. The link-segment model shows these equal and opposite reaction forces. The bone-on-bone force is zero, indicating that the joint articulating surfaces are under neither tension nor compression.
In **case 2**, there is an active contraction of the muscles, so that the total upward force is now 170 N.

The bone-on-bone force is 70N compression. This means that a force of 70N exists across the articulating surfaces.

As far as the lower segment is concerned, there is still a net reaction force of 100N upward (170N upward through muscles and 70N downward across the articulating surfaces).

The lower segment is still acting downward with a force of 100 N; thus, the free-body diagram remains the same.
BASIC LINK-SEGMENT EQUATIONS—THE FREE-BODYDIAGRAM

**Known**

- $a_x, a_y =$ acceleration of segment COM
- $\theta =$ angle of segment in plane of movement
- $\alpha =$ angular acceleration of segment in plane of movement
- $R_{xd}, R_{yd} =$ reaction forces acting at distal end of segment, usually determined from a prior analysis of the proximal forces acting on distal segment
- $M_d =$ net muscle moment acting at distal joint, usually determined from an analysis of the proximal muscle acting on distal segment
**BASIC LINK-SEGMENT EQUATIONS—THE FREE-BODYDIAGRAM**

**Unknown**

- $R_{xp}, R_{yp}$ = reaction forces acting at proximal joint
- $M_p$ = net muscle moment acting on segment at proximal joint

Note that the muscle moment at the proximal end cannot be calculated until the proximal reaction forces $R_{xp}$ and $R_{yp}$ have first been calculated.

1. $\Sigma F_x = ma_x$
   
   \[ R_{xp} - R_{xd} = ma_x \]

2. $\Sigma F_y = ma_y$

   \[ R_{yp} - R_{yd} - mg = ma_y \]

3. About the segment COM, $\Sigma M = I_0 \alpha$
Example

- In a static situation, a person is standing on one foot on a force plate. The ground reaction force is found to act 4 cm anterior to the ankle joint.
- Note that convention has the ground reaction force $R_{y1}$ always acting upward.
- We also show the horizontal reaction force $R_{x1}$ to be acting in the positive direction (to the right). If this force actually acts to the left, it will be recorded as a negative number.
- The subject’s mass is 60 kg, and the mass of the foot is 0.9 kg.
- Calculate the joint reaction forces and net muscle moment at the ankle.
  
  $R_{y1} = \text{body weight} = 60 \times 9.8 = 588 \text{ N.}$
Example

- $\Sigma F_x = m a_x$, $R_{x2} + R_{x1} = m a_x = 0$ (Subject is static)
- $\Sigma F_y = m a_y$, $R_{y2} + R_{y1} = m g = m a_y$
  
  \[
  R_{y2} + 588 - 0.9 \times 9.8 = 0 \quad R_{y2} = -579.2 \text{N}
  \]

- The negative sign means that the force acting on the foot at the ankle joint acts downward.

- About the COM, $\Sigma M = I_0 \alpha$,
  \[
  M_2 - R_{y1} \times 0.02 - R_{y2} \times 0.06 = 0
  \]
  \[
  M_2 = 588 \times 0.02 + (-579.2 \times 0.06) = -22.99 \text{N} \cdot \text{m}
  \]

- The negative sign means that the real direction of the muscle moment acting on the foot at the ankle joint is clockwise, which means that the plantarflexors are active at the ankle joint to maintain the static position.
Example

- From the data collected during the swing of the foot, calculate the muscle moment and reaction forces at the ankle.
- The subject’s mass was 80 kg and the ankle-metatarsal length was 20.0 cm.
- The inertial characteristics of the foot are calculated:
  
  \[
  m = 0.0145 \times 80 = 1.16 \text{ kg}
  \]
  
  \[
  \rho_0 = 0.475 \times 0.20 = 0.095 \text{m}
  \]
  
  \[
  I_0 = 1.16(0.095)^2 = 0.0105 \text{ kg} \cdot \text{m}^2
  \]
  
  \[
  \alpha = 21.69 \text{ rad/s}^2
  \]
Three unknowns, \( R_{x1}, R_{y1}, \) and \( M_1 \), are to be calculated assuming a positive direction as shown.

1. \( \Sigma F_x = ma_x, \)

\[
R_{x1} = 1.16 \times 9.07 = 10.52 \text{ N}
\]

2. \( \Sigma F_y = ma_y, \)

\[
R_{y1} - 1.16g = m(-6.62)
\]

\[
R_{y1} = 1.16 \times 9.8 - 1.16 \times 6.62 = 3.69 \text{ N}
\]

3. At the COM of the foot, \( \Sigma M = l_0 \alpha, \)

\[
M_1 - R_{x1} \times 0.0985 - R_{y1} \times 0.0195 = 0.0105 \times 21.69
\]

\[
M_1 = 0.0105 \times 21.69 + 10.52 \times 0.0985 + 3.69 \times 0.0195
\]

\[
= 0.23 + 1.04 + 0.07 = 1.34 \text{ N} \cdot \text{m}
\]
1. The horizontal reaction force of 10.52N at the ankle is the cause of the horizontal acceleration that we calculated for the foot.

2. The foot is decelerating its upward rise at the end of lift-off. Thus, the vertical reaction force at the ankle is somewhat less than the static gravitational force.

3. The ankle muscle moment is positive, indicating net dorsiflexor activity (tibialis anterior), and most of this moment (1.04 out of 1.34N · m) is required to cause the horizontal acceleration of the foot’s center of gravity, with very little needed (0.23N · m) to angularly accelerate the low moment of inertia of the foot.

\[
\begin{align*}
\sum F_x &= m\alpha_x, \\
R_{xl} &= 1.16 \times 9.07 = 10.52 \text{ N} \\
\sum F_y &= m\alpha_y, \\
R_{yl} - 1.16g &= m(-6.62) \\
R_{yl} &= 1.16 \times 9.8 - 1.16 \times 6.62 = 3.69 \text{ N} \\
3. & \text{ At the COM of the foot, } \sum M = I_0\alpha, \\
M_1 - R_{xl} \times 0.0985 - R_{yl} \times 0.0195 &= 0.0105 \times 21.69 \\
M_1 &= 0.0105 \times 21.69 + 10.52 \times 0.0985 + 3.69 \times 0.0195 \\
&= 0.23 + 1.04 + 0.07 = 1.34 \text{ N} \cdot \text{m}
\end{align*}
\]
Example

- For the same instant in time, calculate the muscle moments and reaction forces at the knee joint. The leg segment was 43.5 cm long.

\[
m = 0.0465 \times 80 = 3.72 \text{ kg}
\]
\[
\rho_0 = 0.302 \times 0.435 = 0.131 \text{ m}
\]
\[
I_0 = 3.72(0.131)^2 = 0.0638 \text{ kg} \cdot \text{m}^2
\]
\[
\alpha = 36.9 \text{ rad/s}^2
\]

- From previous example,

\[
R_{x1} = 10.52 \text{ N}, \quad R_{y1} = 3.69 \text{ N}, \quad \text{and} \quad M_1 = 1.34 \text{ N} \cdot \text{m}.
\]
Example

1. \[ \Sigma F_x = ma_x, \]
   \[ R_{x2} - R_{x1} = ma_x \]
   \[ R_{x2} = 10.52 + 3.72(-0.03) = 10.41 \text{ N} \]

2. \[ \Sigma F_y = ma_y, \]
   \[ R_{y2} - R_{y1} - mg = ma_y \]
   \[ R_{y2} = 3.69 + 3.72 \times 9.8 + 3.72(-4.21) = 24.48 \text{ N} \]

3. About the COM of the leg, \[ \Sigma M = I \alpha, \]
   \[ M_2 - M_1 - 0.169R_{x1} + 0.185R_{y1} - 0.129R_{x2} + 0.142R_{y2} = I \alpha \]
   \[ M_2 = 1.34 + 0.169 \times 10.52 - 0.185 \times 3.69 + 0.129 \times 10.41 \\
      - 0.142 \times 24.48 + 0.0638 \times 36.9 \]
   \[ = 1.34 + 1.78 - 0.68 + 1.34 - 3.48 + 2.35 = 2.65 \text{ N} \cdot \text{m} \]
Discussion

1. $M_2$ is positive. This represents a counterclockwise (extensor) moment acting at the knee. The quadricep muscles at this time are rapidly extending the swinging leg.
2. The angular acceleration of the leg is the net result of two reaction forces and one muscle moment acting at each end of the segment. Thus, there may not be a single primary force causing the movement we observe. In this case, each force and moment had a significant influence on the final acceleration.

$$R_{x2} = 10.52 + 3.72(-0.03) = 10.41 \text{ N}$$
$$R_{y2} = 3.69 + 3.72 \times 9.8 + 3.72(-4.21) = 24.48 \text{ N}$$
$$M_2 = 1.34 + 1.78 - 0.68 + 1.34 - 3.48 + 2.35 = 2.65 \text{ N} \cdot \text{m}$$
In order to measure the force exerted by the body on an external body or load, we need a suitable force-measuring device. Such a device, called a force transducer, gives an electrical signal proportional to the applied force.

- strain gauge type
- Piezoelectric and piezoresistive types

Multidirectional Force Transducers

In order to measure forces in two or more directions, it is necessary to use a bi- or tridirectional force transducer. Such a device is nothing more than two or more force transducers mounted at right angles to each other.
Force Plates

- The most common force acting on the body is the ground reaction force, which acts on the foot during standing, walking, or running.

- This force vector is three-dimensional and consists of a vertical component plus two shear components acting along the force plate surface. These shear forces are usually resolved into anterior—posterior and medial—lateral directions.

- The fourth variable needed is the location of the center of pressure of this ground reaction vector. The foot is supported over a varying surface area with different pressures at each part.

- Even if we knew the individual pressures under every part of the foot, we would be faced with the expensive problem of calculating the net effect of all these pressures as they change with time.
Flat Plate Supported By Four Triaxial Transducers

- It is composed of four triaxial transducers located at (0,0), (0, Z), (X, 0), and (X, Z).
- The location of the center of pressure is determined by the relative vertical forces seen at each of these corner transducers.
- If we designate the vertical forces as $F_{00}$, $F_{X0}$, $F_{0Z}$, and $F_{XZ}$, the total vertical force is $F_Y = F_{00} + F_{X0} + F_{0Z} + F_{XZ}$.
- If all four forces are equal, the COP is at the exact center of the force plate, at $(X/2, Z/2)$. 
Flat Plate Supported By Four Triaxial Transducers

Generalized as

\[
x = \frac{X}{2} \left[ 1 + \frac{(F_{X_0} + F_{XZ}) - (F_{00} + F_{0Z})}{F_Y} \right]
\]

\[
z = \frac{Z}{2} \left[ 1 + \frac{(F_{0Z} + F_{XZ}) - (F_{00} + F_{X0})}{F_Y} \right]
\]

\[FY = F_{00} + F_{X0} + F_{0Z} + F_{XZ}\]
A second type of force plate has one centrally instrumented pillar that supports an upper flat plate.

- Figure shows the forces that act on this instrumented support.
- The action force of the foot $F_y$ acts downward, and the anterior-posterior shear force can act either forward or backward. Consider a reverse shear force $F_x$, as shown. If we sum the moments acting about the central axis of the support, we get:

$$M_z - F_y \cdot x + F_x \cdot y_0 = 0$$

$$x = \frac{F_x \cdot y_0 + M_z}{F_y}$$

- $M_z =$ bending moment about axis of rotation of support
- $y_0 =$ distance from support axis to force plate surface

$F_x$, $F_y$, and $M_z$ continuously change with time, $x$ can be calculated to show how the COP moves across the force plate.
rapid rise at heel contact
knee flexes during midstance
pushoff by ankle muscles

Force plate reaction $F_x$ is backward
$F_x$ is acting forward
The COP starts at the heel, assuming that initial contact is made by the heel, and then progresses forward toward the ball and toe.

The position of the COP relative to the foot cannot be obtained from the force plate data.

The centers of pressure Xcp and Ycp are calculated in absolute coordinates to match those given in the kinematics listing. Ycp was set at 0 to indicate ground level.

For an N-segment system the reaction forces in the x and y direction are:

\[ F_x = \sum_{i=1}^{N} m_i a_{xi} \]
\[ F_y = \sum_{i=1}^{N} m_i (a_{yi} + g) \]

- \( m_i \) = mass of i-th segment
- \( a_{xi} \) = acceleration of i-th segment COM in the x direction
- \( a_{yi} \) = acceleration of i-th segment COM in the y direction
- \( g \) = acceleration due to gravity

Accelerations of the COMs of all segments should be analyzed by using measured reaction force.
Special Pressure-Measuring Sensory Systems

- The COP measured by the force plate is a weighted average of the distributed COPs under the foot that is in contact.
- It does not give pressure at any of the contact points under the foot.
- For example, during midstance in walking or running, there are two main pressure areas: the ball of the foot and the heel, but the force plate records the COP as being under the arch of the foot where there may in fact be negligible pressure.
- To get some insight into the distributed pressures around all contact points, a number of special pressure measurement systems have been developed.
Two thin flexible polyester sheets have electrically conductive electrodes deposited in rows and columns. A thin semiconductive coating is applied between the conductive rows and columns, and its electrical resistance changes with the pressure applied.

(5mm × 5 mm),
F-Scan™ System
Synchronization of Force Plate and Kinematic Data

- Because kinematic data are coming from a completely separate system, there may be problems in time synchronization with the ground reaction data.
- Most optoelectric systems have synchronizing pulses that must be recorded simultaneously with the force records.
- Similarly, TV systems must generate a pulse for each TV field that can be used to synchronize with the force signals.
- The major imaging system that has problems is cine. Here, the movie cameras must have a sync pulse generated every frame, and somehow the number of the pulse must be made available to the person doing the film digitization.
For a subject in late stance (see Figure 5.13), during pushoff the following foot accelerations were recorded:
- $a_x = 3.25 \text{ m/s}^2$,
- $a_y = 1.78 \text{ m/s}^2$,
- $\alpha = -45.35 \text{ rad/s}^2$.
- The mass of the foot is 1.12kg
- The moment of inertia is 0.01 kg·m$^2$

Calculate the reaction forces and the moment at the ankle
For a subject in late stance (see Figure 5.13), during pushoff the following foot accelerations were recorded:
- $a_x = 3.25 \text{ m/s}^2$
- $a_y = 1.78 \text{ m/s}^2$
- $\alpha = -45.35 \text{ rad/s}^2$
- Mass $= 1.12 \text{ kg}$
- $I_0 = 0.01 \text{ kg} \cdot \text{m}^2$

The polarity and the magnitude of this ankle moment indicate strong plantarflexor activity acting to push off the foot and cause it to rotate clockwise about the metatarsophalangeal joint.

\[ F_{ax} + F_x = ma_x \]
\[ F_{ax} = 1.12 \times 3.25 - 160.25 = -156.6 \text{ N} \]
\[ F_{ay} + F_y - mg = ma_y \]
\[ F_{ay} = 1.12 \times 1.78 - 765.96 + 1.12 \times 9.81 = -753.0 \text{ N} \]

About the center of mass of the foot, $\Sigma M = I\alpha$,
\[ M_a + F_x \times 0.084 + F_y \times 0.079 - F_{ay} \times 0.056 - F_{ax} \times 0.076 = 0.01(-45.35) \]
\[ M_a = -0.01 \times 45.35 - 0.084 \times 160.25 - 0.079 \times 765.96 - 0.056 \times 753.0 - 0.076 \times 156.6 = -128.5 \text{ N} \cdot \text{m} \]
Interpretation of Moment-of-Force Curves

- Counterclockwise moments acting on a segment distal to the joint are positive.
- A plantarflexor moment (acting on its distal segment) is negative,
- A knee extensor moment is shown to be positive,
- A hip extensor moment is negative

\[ M_s = M_k - M_a - M_h \]

- Hip extensors are active
- Hip flexors are active

Pushoff to toe-off

- Hip flexors are active
- Hip extensors are active

- A knee extensor moment is shown to be positive,
Differences between Center of Mass and Center of Pressure

- Time 1 has the body’s COM (shown by the vertical body weight vector, W) to be ahead of the COP (shown by the vertical ground reaction vector, R).
- At Time 1, \( W_g > R_p \), the body will experience a clockwise angular acceleration, \( \alpha \).
- In order to correct this forward imbalance, the subject will increase his or her lantarflexor activity, which will increase the COP such that at Time 2 the COP will be anterior of the COM.
- Now \( R_p > W_g \). Thus, \( \alpha \) will reverse and will start to decrease \( \omega \) until, at Time 3
Differences between Center of Mass and Center of Pressure

- at Time 3, the time integral of $\alpha$ will result in a reversal of $\omega$. Now both $\omega$ and $\alpha$ are counterclockwise, and the body will be experiencing a backward sway.
- at Time 4 is subject decreases his or her COP by reduced plantarflexor activation.
- Now $W_g > R_p$ and $\alpha$ will reverse, and, after a period of time, $\omega$ will decrease and reverse.
- at Time 5 the body will return to the original conditions, as seen.
a typical 40-s record of the center of pressure (COPx ) and center of mass (COMx ) in the anterior/posterior direction of an adult subject standing quietly. Note that both signals are virtually in phase and that COP is slightly greater than COM.

COP must move ahead of and behind the COM in order to decelerate it and reverse its direction. Note that all reversals of direction of COM coincide with an overshoot of the COP signal.
There is a common model that allows us to analyze the dynamics of balance: the inverted pendulum model, which relates the trajectories of the COP and COM.

In the sagittal plane, assuming that the body swayed about the ankles:

\[
\text{COP}_x - \text{COM}_x = -\frac{I_s C\bar{OM}_x}{Wh}
\]

- \(I_s\) = moment of inertia of body about ankles in sagittal plane
- \(C\bar{OM}_x\) = forward acceleration of COM
- \(W\) = body weight above ankles
- \(h\) = height of COM above ankles

In the frontal plane, the balance equation is virtually the same:

\[
\text{COP}_z - \text{COM}_z = -\frac{I_f C\bar{OM}_z}{Wh}
\]

- \(I_f\) = moment of inertia of body about ankles in frontal plane
- \(C\bar{OM}_z\) = medial/lateral acceleration of COM

\[
\text{COP}_x = \frac{Ma}{R}, \text{ where } Ma \text{ is the sum of the right and left plantarflexor moments and } R \text{ is the total vertical reaction force at the ankles.}
\]

\[
\text{COP}_z = \frac{Mt}{R}, \text{ where } Mt = Mal + Mar + Mhl + Mhr \text{ where } Mal \text{ with } Mar \text{ are the left and right frontal ankle moments, while } Mhl \text{ and } Mhr \text{ are the left and right frontal plane hip moments}
\]